Çankaya University Journal of Science and Engineering
Volume 13, No. 2 (2016) 011–017

On Graded 2-Absorbing and Graded Weakly 2-Absorbing Ideals of a Commutative Ring

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Abstract: Let G be a group with identity e and R be a G-graded commutative ring with $1 \neq 0$. In this paper, we study the graded versions of 2-absorbing and weakly 2-absorbing ideals which are generalizations of the graded prime and graded weakly prime ideals, respectively. A graded proper ideal I of R is called a graded 2-absorbing (resp. graded weakly 2-absorbing) ideal if whenever $abc \in I$ (resp. $0 \neq abc \in I$) for homogeneous elements $a,b,c \in R$, then $ab \in I$ or $ac \in I$ or $bc \in I$. It is clear that a graded ideal which is a 2-absorbing ideal, is a graded 2-absorbing ideal, but we show that the converse is not true in general. It is proved that if $I = \oplus_{g\in G} I_g$ is a graded weakly 2-absorbing ideal of R, then either I is a 2-absorbing ideal of R or $I_g^3 = (0)$ for all $g \in G$. It is also shown that if $I = \oplus_{i\in G} I_g$ is a graded weakly 2-absorbing ideal of R, then for each $g \in G$, either I_g is a 2-absorbing R_e-submodule of R_g or $(I_g :_{R_e} R_g)^2 I_g = 0$.

Keywords: Graded prime ideal, graded 2-absorbing ideal, graded weakly 2-absorbing ideal.

1. Introduction

Throughout, all rings are assumed to be commutative rings with a non-zero identity. In recent years, various generalizations of prime ideals have been studied by several authors (see, for example, [1, 2, 8]). Anderson and Smith in [2], defined a weakly prime ideal, i.e., a proper ideal P of R with the property that for $a,b \in R$, $0 \neq ab \in P$ implies $a \in P$ or $b \in P$. The concept of 2-absorbing ideal which is a generalization of prime ideal, has been introduced and investigated by A. Badawi in [5] and studied in [3, 5, 7, 9]. As in [5] (resp. [6]), a proper ideal I of a commutative ring R is called a 2-absorbing (resp. weakly 2-absorbing) ideal if whenever $abc \in I$ (resp. $0 \neq abc \in I$) for $a,b,c \in R$, then $ab \in I$ or $ac \in I$ or $bc \in I$. Also, graded prime ideals and graded weakly prime ideals in a commutative graded ring have been studied in [14] and [4] respectively. In this paper, we introduce the graded 2-absorbing and graded weakly 2-absorbing ideals which are the graded versions of 2-absorbing and weakly 2-absorbing ideals on the one hand and generalizations of graded prime and graded weakly prime ideals on the other.

Before we state our results let us recall some notation and terminology. Let G be a group with identity e. A G-graded ring is a ring R together with a decomposition $R = \oplus_{g\in G} R_g$ (as a $\mathbb{Z}$-module)

ISSN 1309 - 6788

such that $R_g R_h \subseteq R_{g+h}$ for all $g, h \in G$. We denote this ring by $G(R)$. The summands R_g are called homogeneous components and the elements of these summands are called homogeneous elements of degree g. The set of all homogeneous elements of R is denoted by $h(R) = \bigcup_{g\in G} R_g$. If $a \in R$, then a can be written uniquely as $a = \sum_{g\in G} a_g$ and a_g is called the g-th homogeneous component of a. Moreover, if $R = \oplus_{g\in G} R_g$ is a graded ring, then R_e is a subring of R and R_g is an R_e-module for all $g \in G$.

An ideal I of a graded ring $R = \oplus_{g\in G} R_g$ is said to be a graded (or homogeneous) ideal, whenever $I = \oplus_{g\in G}(I \cap R_g)$. Equivalently, I is a graded ideal of R if for every element $a \in I$ all homogeneous components of a are in I. A graded ideal P of $G(R)$ is said to be a graded (resp., graded weakly) prime ideal if $P \neq R$ and whenever $ab \in P$ (resp., $0 \neq ab \in P$) for $a, b \in h(R)$, then $a \in P$ or $b \in P$. For other notaions and terminology about graded rings, we refer to [12].

Definition 1. A graded ideal I of $G(R)$ is said to be a graded (resp. graded weakly) 2-absorbing ideal if $I \neq R$ and whenever $abc \in I$ (resp. $0 \neq abc \in I$) for $a, b, c \in h(R)$, then $ab \in I$ or $ac \in I$ or $bc \in I$.

It is well-known that a proper graded ideal P of $G(R)$ is a prime ideal if and only if it is a graded prime ideal of R (see [13, Proposition 1]). However, we show that a graded 2-absorbing ideal of a graded ring R need not be a 2-absorbing ideal of R (Example 2.1). Clearly, every graded 2-absorbing ideal of a graded ring R is a graded weakly 2-absorbing ideal, but the converse is not true in general (Remark 2.1). It is shown that if $I = \oplus_{g\in G} I_g$ is a graded weakly 2-absorbing ideal of R, then either I is a 2-absorbing ideal of R or $I_g^3 = (0)$ for all $g \in G$ (Theorem 1). Let M be an R-module. A proper submodule N of M is called 2-absorbing if whenever $r, s \in R$, $m \in M$ and $rsm \in N$, then $rm \in N$ or $sm \in N$ or $rs \in (N : M) = \{r \in R \mid rM \subseteq N\}$ [11]. It is shown that if $I = \oplus_{g\in G} I_g$ is a graded weakly 2-absorbing ideal of R, then for each $g \in G$, either I_g is a 2-absorbing submodule of the R_e-module R_g or $(I_g :_{R_e} R_g)^2 I_g = 0$ (Theorem 7).

2. Graded 2-Absorbing and Graded Weakly 2-Absorbing Ideals

Recall that a proper ideal P of a graded ring R is a graded (resp., graded weakly) prime ideal of R if whenever $ab \in P$ (resp., $0 \neq ab \in P$) for $a, b \in h(R)$, then $a \in P$ or $b \in P$. By [13, Proposition 1], if P is a proper graded ideal of a graded ring R, then P is a prime ideal of R if and only if it is a graded prime ideal of R. In the following example, we see that this not true when "prime" is replaced by "2-absorbing".

Example 2.1. Let $R = \mathbb{Z}[x, y]$ and $I = (6, 2x, 2y, xy)$. Consider the standard grading on R. Then, I is a graded 2-absorbing ideal of R which is not a 2-absorbing ideal. To see this, let $f_1 = 3$, $f_2 = x + 2$ and $f_3 = y + 2$, then $f_1 f_2 f_3 \in I$, but no product of any 2 of the f_i's is in I. Thus I is not a 2-absorbing ideal of R. Now we show that I is a graded 2-absorbing ideal of R. For this purpose, let $f_1 = \sum_{i+j=m} a_{ij}x^iy^j$, $f_2 = \sum_{i+j=n} b_{ij}x^iy^j$ and $f_3 = \sum_{i+j=s} c_{ij}x^iy^j$ be homogeneous elements of R (of degree m,n and s, respectively) such that $f_1f_2f_3 \in I$. We can write $f_1 = a_{m0}x^m + a_{0m}y^m + P(xy)$, $f_2 = b_{n0}x^n + b_{0n}y^n + Q(xy)$ and $f_3 = c_{s0}x^s + c_{0s}y^s + T(xy)$. Thus

$$f_1f_2f_3 = \left(a_{m0}x^m + a_{0m}y^m + P(xy)\right)\left(b_{n0}x^n + b_{0n}y^n + Q(xy)\right)\left(c_{s0}x^s + c_{0s}y^s + T(xy)\right) \in I.$$

This implies that $a_{m0}b_{n0}c_{s0}x^{m+n+s} + a_{0m}b_{0n}c_{0s}y^{m+n+s} \in I$. Thus $2|a_{m0}b_{n0}c_{s0}$ and $2|a_{0m}b_{0n}c_{0s}$. We consider the following cases:

(1) If $2|a_{m0}$ and $2|a_{0m}$. Then, $f_1 \in I$.
(2) If $2|a_{m0}$ and $2|b_{0n}$. Then, $f_1f_2 \in I$.
(3) If $2|a_{m0}$ and $2|c_{0s}$. Then, $f_1f_3 \in I$.

The other cases are similar. Hence I is a graded 2-absorbing ideal of R which is not a 2-absorbing ideal.

Remark 2.1. It is clear that every graded 2-absorbing ideal of $G(R)$ is a graded weakly 2-absorbing ideal of R, but the converse is not true in general. For instance, let $R = \mathbb{Z}_{12}$ and consider the trivial grading on R. Clearly $I = (0)$ is a graded weakly 2-absorbing ideal of R, however it is not a graded 2-absorbing ideal of R since $\overline{2}.\overline{2}.\overline{3} \in I$, but $\overline{2}.\overline{2} \notin I$ and $\overline{2}.\overline{3} \notin I$. The following theorem gives a sufficient condition for a graded weakly 2-absorbing ideal to be a graded 2-absorbing ideal.

Theorem 1. Let $I = \oplus_{g\in G} I_g$ be a graded weakly 2-absorbing ideal of $G(R)$. Then, either I is a graded 2-absorbing ideal of R or $I_g^3 = (0)$ for all $g \in G$.

Proof. Let $I_g^3 \neq 0$ for some $g \in G$. Let $a,b,c \in h(R)$ such that $abc \in I$. If $abc \neq 0$, since I is a graded weakly 2-absorbing ideal, we have $ab \in I$ or $ac \in I$ or $bc \in I$. So let $abc = 0$. If $abI_g \neq (0)$, then there exists $d \in I_g$ such that $abd \neq 0$. Thus $0 \neq abd = ab(d+c) \in I$. Hence, either $ab \in I$ or $b(d+c) \in I$ or $a(d+c) \in I$. As $d \in I$, we have either $ab \in I$ or $bc \in I$ or $ac \in I$. So, we can assume that $abI_g = acI_g = bcI_g = (0)$.
If $aI_g^2 \neq 0$, then there are $d,e \in I_g$ such that $ade \neq 0$. Thus $0 \neq ade = a(b+d)(c+e) \in I$. Hence either $a(b+d) \in I$ or $a(c+e) \in I$ or $(b+d)(c+e) \in I$. This implies that either $ab \in I$ or $ac \in I$ or $bc \in I$, since $d,e \in I$. So, we can also assume that $aI_g^2 = bI_g^2 = cI_g^2 = 0$.
Since $I_g^3 \neq (0)$, there exist $d,e,f \in I_g$ such that $def \neq 0$. Then,

$$(a+d)(b+e)(c+f) = def \in I.$$

Since $def \neq 0$ and I is a graded weakly 2-absorbing ideal of R, we may assume without loss of generality that $(a+d)(b+e) \in I$, and therefore $ab \in I$ (note that $ae,db,de \in I$). Hence I is a graded 2-absorbing ideal of R. ■

The graded radical of a graded ideal I of $G(R)$, denoted $\mathrm{Grad}(I)$, is the set of all $x \in R$ such that for each $g \in G$, there exists $n_g > 0$ with $x_g^{n_g} \in I$.

Corollary 1. Let $I = \oplus_{g\in G} I_g$ be a graded weakly 2-absorbing ideal which is not a graded 2-absorbing ideal of $G(R)$. Then, $\mathrm{Grad}(I) = \mathrm{Grad}(0)$.

Proof. It suffices to show that $\mathrm{Grad}(I) \subseteq \mathrm{Grad}(0)$. Let $a \in I$. By Theorem 1, $I_g^3 = (0)$ for all $g \in G$, and so $a \in \mathrm{Grad}(0)$; hence $I \subseteq \mathrm{Grad}(0)$. From [14, Propositon 1.2], it follows that $\mathrm{Grad}(I) \subseteq \mathrm{Grad}(0)$, as required. ■

Lemma 1. Let $J \subseteq I$ be graded ideals of $G(R)$ with $I \neq R$. Then, the following hold:

(1) If I is a graded weakly 2-absorbing ideal, then I/J is a graded weakly 2-absorbing ideal of R/J.
(2) If J and I/J are graded weakly 2-absorbing ideals, then I is a graded weakly 2-absorbing ideal.

Proof. (1) Straightforward.

(2) Let $0 \neq abc \in I$ for some $a,b,c \in h(R)$. Thus $(a+J)(b+J)(c+J) \in I/J$. If $abc \in J$, then we have either $ab \in J \subseteq I$ or $ac \in J \subseteq I$ or $bc \in J \subseteq I$, since J is a graded weakly 2-absorbing ideal. So, let $abc \notin J$. Then, $0 \neq (a+J)(b+J)(c+J) \in I/J$. Since I/J is a graded weakly 2-absorbing ideal of R/J, it follows that either $(a+J)(b+J) \in I/J$ or $(a+J)(c+J) \in I/J$ or $(b+J)(c+J) \in I/J$. Therefore $ab \in I$ or $ac \in I$ or $bc \in I$.

■

Theorem 2. Let $I = \oplus_{g\in G} I_g$ and $J = \oplus_{g\in G} J_g$ be graded weakly 2-absorbing ideals of $G(R)$ which are not graded 2-absorbing ideals. Then, $I+J$ is a graded weakly 2-absorbing ideal of $G(R)$.

Proof. By Corollary 1, we have $\mathrm{Grad}(I)+\mathrm{Grad}(J) = \mathrm{Grad}(0) \neq R$. Thus $I+J$ is a proper ideal of R. Since $(I+J)/J \cong J/(I\cap J)$ and J is a graded weakly 2-absorbing ideal of R, by Lemma 1 (1), $(I+J)/J$ is a graded weakly 2-absorbing ideal of R/J, and then the assertion follows from Lemma 1 (2). ■

Let M be an R-module. A proper submodule N of M is called a 2-absorbing (resp., weakly 2-absorbing) submodule if whenever $a,b \in R$ and $m \in M$ with $abm \in N$ (resp., $0 \neq abm \in N$), then $ab \in (N :_R M)$ or $am \in N$ or $bm \in N$. In [11, Proposition 1], it has been proved that if N is a 2-absorbing submodule of an R-module M, then $(N :_R M)$ is a 2-absorbing ideal of R. This may be compared with the following result in the weakly 2-absorbing setting.

Theorem 3. Let N be a weakly 2-absorbing submodule of a torsion free R-module M. Then, $(N :_R M)$ is a weakly 2-absorbing ideal of R.

Proof. Clearly $(N :_R M) \neq R$. Let $0 \neq abc \in (N :_R M)$ and $ac, bc \notin (N :_R M)$ for $a, b, c \in R$. Then, there are $x_1, x_2 \in M$ such that $acx_1 \in M \setminus N$ and $bcx_2 \in M \setminus N$. If $0 \neq abc(x_1 + x_2) \in N$, then since N is a weakly 2-absorbing submodule of M, we have $ab \in (N :_R M)$ or $ac(x_1 + x_2) \in N$ or $bc(x_1 + x_2) \in N$. If $ab \in (N :_R M)$, then there is nothing to prove. Therefore, let $ac(x_1 + x_2) \in N$. Then, $acx_2 \notin N$, since $acx_2 \in N$ implies that $acx_1 \in N$ which is a contradiction. Thus $acx_2 \notin N$ and $bcx_2 \notin N$ while $0 \neq abcx_2 \in N$ (it is clear that $abcx_2 \neq 0$ since M is torsion free and $bcx_2 \notin N$). Hence, $ab \in (N :_R M)$. By a similar argument as above one can easily show that if $bc(x_1 + x_2) \in N$, then $ab \in (N :_R M)$. Therefore, let $abc(x_1 + x_2) = 0$. Since M is a torsion free R-module, this implies that $x_2 = -x_1$. Thus $0 \neq abcx_1 \in N$, $acx_1 \notin N$ and $bcx_1 = -bcx_2 \notin N$. Since N is a weakly 2-absorbing submodule of M, this implies that $ab \in (N :_R M)$. Hence, $(N :_R M)$ is a weakly 2-absorbing ideal of R. ■

Theorem 4. Let $I = \oplus_{g \in G} I_g$ be a graded (resp. graded weakly) 2-absorbing ideal of $G(R)$. Then, for each $g \in G$, I_g is a 2-absorbing (resp. weakly 2-absorbing) R_e-submodule of R_g. Moreover, $(I_g :_{R_e} R_g)$ is a 2-absorbing (resp. weakly 2-absorbing) ideal of R_e for all g.

Proof. Let $abr \in I_g \subseteq I$ (resp. $0 \neq abr \in I_g \subseteq I$) for $a, b \in R_e$ and $r \in R_g$. Since I is a graded 2-absorbing (resp., graded weakly 2-absorbing) ideal of R, it follows that $ab \in I$ or $ar \in I$ or $br \in I$. This implies that $ab \in (I_g :_{R_e} R_g)$ or $ar \in I \cap R_g = I_g$ or $br \in I \cap R_g = I_g$. Thus I_g is a 2-absorbing (resp., weakly 2-absorbing) R_e-submodule of R_g.

The “moreover” statement immediately follows from [11, Proposition 1] and Theorems 3. ■

Definition 2. Let $I = \oplus_{g \in G} I_g$ be a graded ideal of $G(R)$. We say that the subgroup I_g of R_g is a 2-absorbing subgroup of R_g, if $I_g \neq R_g$ and whenever $a, b, c \in h(R)$ with $abc \in I_g$, then either $ab \in I_g$ or $bc \in I_g$ or $ac \in I_g$.

Proposition 1. Let $I = \oplus_{g \in G} I_g$ be a graded ideal of $G(R)$. If for all $g \in G$, I_g is a 2-absorbing subgroup of R_g, then I is a graded 2-absorbing ideal of R.

Proof. Let $abc \in I$ for $a, b, c \in h(R)$. Then, $abc \in I_g$ for some $g \in G$. Hence, $ab \in I_g$ or $ac \in I_g$ or $bc \in I_g$, since I_g is a 2-absorbing subgroup of R_g. This completes the proof. ■

Theorem 5. Let $I = \oplus_{g \in G} I_g$ be a graded 2-absorbing ideal of $G(R)$ and $g \in G$. If $a, b \in R_g$ such that $ab \notin I$, then $(I_{2g} :_{R_e} ab) = (I_g :_{R_e} a) \cup (I_g :_{R_e} b)$.

Proof. Let $c \in (I_g :_{R_e} a) \cup (I_g :_{R_e} b)$. Then, we may assume without loss of generality that $c \in (I_g :_{R_e} a)$. Thus $ca \in I_g \subseteq I$ and therefore $cab \in I \cap R_{2g} = I_{2g}$. Hence, $c \in (I_{2g} :_{R_e} ab)$. For the reverse inclusion, assume that $c \in (I_{2g} :_{R_e} ab)$. Then, $cab \in I_{2g} \subseteq I$. Since I is a graded 2-absorbing ideal and $ab \notin I$, we conclude that either $ca \in I$ or $cb \in I$. Thus $ca \in I \cap R_g = I_g$ or $cb \in I \cap R_g = I_g$ and therefore $c \in (I_g :_{R_e} a) \cup (I_g :_{R_e} b)$. ■

Theorem 6. Let $I = \oplus_{g\in G} I_g$ be a graded weakly 2-absorbing ideal of R and $g \in G$. Then, for each $a, b \in R_g$ with $ab \notin I$,

$$(I_g :_{R_e} ab) \cup (I_{2g} :_{R_e} ab) = (I_g :_{R_e} a) \cup (I_g :_{R_e} b) \cup (0 :_{R_e} ab).$$

Proof. Clearly $(0 :_{R_e} ab) \subseteq (I_g :_{R_e} ab) \cup (I_{2g} :_{R_e} ab)$. Let $c \in (I_g :_{R_e} a) \cup (I_g :_{R_e} b)$. We may assume without loss of generality that $c \in (I_g :_{R_e} a)$. Then, $ca \in I_g \subseteq I$ and therefore $cab \in I \cap R_{2g} = I_{2g}$. Hence, $c \in (I_{2g} :_{R_e} ab)$. For the reverse inclusion, first we show that $(I_g :_{R_e} ab) \subseteq (0 :_{R_e} ab)$. Let $c \in (I_g :_{R_e} ab)$. Then, $cab \in I_g$. On the other hand, $cab \in R_{2g}$. Thus, we must have $cab = 0$, i.e., $c \in (0 :_{R_e} ab)$. Now let $c \in (I_{2g} :_{R_e} ab)$. If $cab = 0$, then $c \in (0 :_{R_e} ab)$, and so we are done. Therefore, let $0 \neq cab \in I_{2g} \subseteq I$. Since I is a graded weakly 2-absorbing ideal and $ab \notin I$, we have either $ca \in I$ or $cb \in I$. Thus $ca \in I \cap R_g = I_g$ or $cb \in I \cap R_g = I_g$, i.e., $c \in (I_g :_{R_e} a) \cup (I_g :_{R_e} b)$. This completes the proof. ■

Theorem 7. Let $I = \oplus_{g\in G} I_g$ be a graded weakly 2-absorbing ideal of $G(R)$. Then, for each $g \in G$, either I_g is a 2-absorbing R_e-submodule of R_g or $(I_g :_{R_e} R_g)^2 I_g = 0$.

Proof. By Theorem 4, for each $g \in G$, I_g is a weakly 2-absorbing submodule of the R_e-module R_g. Assume that $(I_g :_{R_e} R_g)^2 I_g \neq 0$ for some $g \in G$ and $abr \in I_g$ for $a, b \in R_e$ and $r \in R_g$. If $abr \neq 0$, then $ar \in I_g$ or $br \in I_g$ or $ab \in (I_g :_{R_e} R_g)$, since I_g is a weakly 2-absorbing R_e-submodule of R_g. Therefore, let $abr = 0$. If $abI_g \neq 0$, then there exists $c \in I_g$ such that $abc \neq 0$. Thus $0 \neq abc = ab(c+r) \in I_g$. It follows that $a(c+r) \in I_g$ or $b(c+r) \in I_g$ or $ab \in (I_g :_{R_e} R_g)$. Since $c \in I_g$, we conclude that $ar \in I_g$ or $br \in I_g$ or $ab \in (I_g :_{R_e} R_g)$. Therefore, let $abI_g = 0$. If $ar(I_g :_{R_e} R_g) \neq 0$, then there exists $c \in (I_g :_{R_e} R_g)$ such that $arc \neq 0$. Thus $0 \neq arc = ar(c+b) \in I_g$ and hence $a(c+b) \in (I_g :_{R_e} R_g)$ or $ar \in I_g$ or $r(c+b) \in I_g$. Thus $ab \in (I_g :_{R_e} R_g)$ or $ar \in I_g$ or $br \in I_g$ (note that $c \in (I_g :_{R_e} R_g)$ implies that $cr \in I_g$). So we may also assume that $ar(I_g :_{R_e} R_g) = br(I_g :_{R_e} R_g) = 0$. If $aI_g(I_g :_{R_e} R_g) \neq 0$, then there are $d \in (I_g :_{R_e} R_g)$ and $e \in I_g$ such that $ade \neq 0$. Since $0 \neq ade = a(b+d)(r+e) \in I_g$, we have $a(b+d) \in (I_g :_{R_e} R_g)$ or $a(r+e) \in I_g$ or $(b+d)(r+e) \in I_g$ and so $ab \in (I_g :_{R_e} R_g)$ or $ar \in I_g$ or $br \in I_g$. Thus we can also assume that $aI_g(I_g :_{R_e} R_g) = bI_g(I_g :_{R_e} R_g) = 0$. Finally we can assume that $r(I_g :_{R_e} R_g)^2 = 0$, for if $r(I_g :_{R_e} R_g)^2 \neq 0$, then $rcc' \neq 0$ for some $c, c' \in (I_g :_{R_e} R_g)$. Thus $0 \neq rcc' = r(a+c)(b+c') \in I_g$ and hence $(a+c)(b+c') \in (I_g :_{R_e} R_g)$ or $r(a+c) \in I_g$ or $r(b+c') \in I_g$. Therefore $ab \in (I_g :_{R_e} R_g)$ or $ar \in I_g$ or $br \in I_g$. Hence, we can assume that $r(I_g :_{R_e} R_g)^2 = 0$. Since $(I_g :_{R_e} R_g)^2 I_g \neq 0$, there are $c, c' \in (I_g :_{R_e} R_g)$ and $d \in I_g$ such that $cc'd \neq 0$. Thus $0 \neq cc'd = (a+c)(b+c')(d+r) \in I$; so we have $(a+c)(b+c') \in (I_g :_{R_e} R_g)$ or $(a+c')(d+r) \in I_g$ or $(b+c')(d+r) \in I_g$, and therefore $ab \in (I_g :_{R_e} R_g)$ or $ar \in I_g$ or $br \in I_g$. Hence, I_g is a 2-absorbing R_e-submodule of R_g. ■

3. Conclusions

In this paper, we have introduced and studied graded 2-absorbing and graded weakly 2-absorbing ideals of a G-graded ring which are generalizations of graded prime ideals and graded weakly prime ideals respectively. Among other results, it has been shown that a graded 2-absorbing ideal $I = \oplus_{g \in G} I_g$ of R is not necessarily a 2-absorbing ideal unless $I_g^3 \neq (0)$ for some $g \in G$.

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